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COMMENT

Electrostatic torque—a misinterpretation

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Abstract
In a work published by Khachatourian and Wistrom, it was shown that a special arrangement of three conducting spheres could expel a mechanic torque on these conductors [1]. They claimed to have experimental data available, which show the validity of their claim [2]. We will demonstrate from basic calculations that this effect does not exist, but is an artefact of truncating a series expansion.

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1. Introduction

In normal conduction matter, it is a matter of fact that the electrostatic field points perpendicular to the conductor surface. In the case of a non-vanishing parallel component to the surface, the electric field acting on the free electrons will move them along the surface till the equilibrium with vanishing tangential electric field is reached. This basic assumption is stressed in each course book on electrostatics (see e.g. [3]). As there are no special symmetries assumed in the above arguments, this consideration has to be valid for any shape and distribution of charged and non-charged conductors.

2. Mathematics

The set-up considered by Khachatourian and Wistrom consisted of three spheres, where one is put to high voltage, and the other two spheres are floating. The two floating spheres are put on torsion bars, in order to allow free rotation of the spheres. The calculation was done using an expansion in legendre polynomials [1]. They mentioned that a two-sphere assembly can be well calculated using cylinder coordinates. Placing a third sphere to the assembly, the cylinder geometry is destroyed and the charge distribution on each sphere is on longer cylinder symmetric. Due to a complicated mathematical procedure inaugurated by Khachatourian and Wistrom [4], they were able to find an analytic expression for this problem. In the last step, Khachatourian and Wistrom stopped the summing of the series after the first term. In order to overcome the problem of truncating a series expansion, we will show that the simple setup

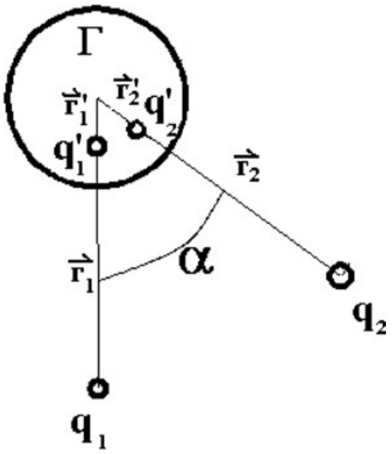

Figure 1. It is claimed by Khachatourian and Wistrom that a torque should act on the sphere Γ due to the two point charges q_1 and q_2.

shown in figure 1 can be calculated using a mirror charge approach. It has to be emphasized that this approach automatically introduces the fact that the electric field is perpendicular to the conductor surface.

The magnitude of the mirror charges q_1' and q_2' in the sphere Γ shown in figure 1 caused by the two point charges q_1 and q_2 is given by

$$q_i' = \frac{-R}{r_i} q_i \tag{1}$$

and is placed at a distance r_1' and r_2' from the centre of the sphere

$$\vec{r}_i' = \frac{R^2}{r_i^2} \vec{r}_i . \tag{2}$$

The forces acting on the sphere are Coulomb forces between the mirror charges q_1' and q_2' and the point charges q_1 and q_2, respectively. From these forces, only the two acting between q_1 and q_2' as well as between q_2' and q_1 cause a torque acting on the sphere. The two forces between charge and mirror charge with the same index are radial to the sphere, and therefore produce no torque. It has to be emphasized that the mirror charges q_1' and q_2 are introduced from the charges q_1 and q_2, and can therefore not move along the conductor. In addition, the torque on the sphere due to the forces between q_1' and q_2' is zero

$$T_{1',2'} = -T_{2',1'} . \tag{3}$$

Forces between the point charges q_1 and q_2 have to be captured by the structure of a possible experiment, and are therefore of no interest in this consideration. The force acting on the point charge q_i' due to the charge q_j is given by

$$F_{i,j} = \frac{1}{4\pi\epsilon_0} \frac{q_i' \cdot q_j}{|\vec{r}_i' - \vec{r}_j|^3} (\vec{r}_i' - \vec{r}_j). \tag{4}$$

The overall torque acting on the sphere Γ is given by

$$\begin{aligned} \vec{T} &= \frac{1}{4\pi\epsilon_0} \left(\vec{r}_1' \times \frac{q_1' \cdot q_2}{|\vec{r}_1' - \vec{r}_2|^3} (\vec{r}_1' - \vec{r}_2) + \vec{r}_2' \times \frac{q_2' \cdot q_1}{|\vec{r}_2' - \vec{r}_1|^3} (\vec{r}_2' - \vec{r}_1) \right) \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{q_1' \cdot q_2}{|\vec{r}_1' - \vec{r}_2|^3} (\vec{r}_2 \times \vec{r}_1') + \frac{q_2' \cdot q_1}{|\vec{r}_2' - \vec{r}_1|^3} (\vec{r}_1 \times \vec{r}_2') \right) . \end{aligned} \tag{5}$$

When inserting 1 and 2 in 5, it can be seen easily that the torque acting on sphere Γ is zero. Therefore, each pair of charges causes a vanishing torque acting on the sphere. This is still true, if one considers not point charges influencing the sphere, but taking real charge distributions (which are in a first approach still build-up from point charges—the limit to a continuous charge distribution is simple, and should not be stressed in much detail).

3. Conclusion

From our simple calculation, we were able to show that the electrostatic torque claimed by Khachatourian and Wistrom does not exist. However, some disturbances might occur in such type of experiment coming from switching on and off the high voltage. During this short time, the non-static conditions might lead to some movement of the spheres. In addition, an imperfect sphere geometry, twisted suspending wires or ambient conditions (turbulences) can cuase some temporary motion. However, due to theoretical calculations it is impossible to get a stationary electrostatic torque acting on conducting surfaces.

References

[1] Khachatourian A V M and Wistrom A O 2003 *J. Phys. A: Math. Gen.* **36** 6495–508
[2] Wistrom A O and Khachatourian A V M 2002 *Appl. Phys. Lett.* **80** 2800–1
[3] Greiner W 1998 *Classical Electrodynamics* (New York: Springer)
[4] Khachatourian A V M and Wistrom A O 2003 *J. Math. Phys.* **44** 849
[5] Khachatourian A V M and Wistrom A O 2003 *J. Phys. A: Math. Gen.* **36** 8359